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Generalized duality between local vector theories in $D = 2 + 1$

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Abstract: The existence of an interpolating master action does not guarantee the same spectrum for the interpolated dual theories. In the specific case of a generalized self-dual (GSD) model defined as the addition of the Maxwell term to the self-dual model in $D = 2 + 1$, previous master actions have furnished a dual gauge theory which is either nonlocal or contains a ghost mode. Here we show that by reducing the Maxwell term to first order by means of an auxiliary field we are able to define a master action which interpolates between the GSD model and a couple of non-interacting Maxwell-Chern-Simons theories of opposite helicities. The presence of an auxiliary field explains the doubling of fields in the dual gauge theory. A generalized duality transformation is defined and both models can be interpreted as self-dual models. Furthermore, it is shown how to obtain the gauge invariant correlators of the non-interacting MCS theories from the correlators of the self-dual field in the GSD model and vice-versa. The derivation of the non-interacting MCS theories from the GSD model, as presented here, works in the opposite direction of the soldering approach.

Keywords: Duality in Gauge Field Theories, Chern-Simons Theories, Field Theories in Lower Dimensions.

Contents

1. Introduction

The existence of different but equivalent descriptions of the same physical theory can help us to reveal deep aspects of the theory which are apparent in one formulation but hidden in the other one. One successful example is the bosonization program in $1+1$ dimensions [1, 2]. Recent examples are provided by the AdS/CFT correspondence [3] and the work of [4] where duality played a key role in a rigorous proof of confinement in a four dimensional theory. A simple approach for deriving dual theories at quantum level is the use of interpolating master actions [5], see [6] for a review. In [5] a first order master action was suggested in order to prove duality equivalence between a non-gauge theory of the self-dual (SD) type [7] (first order) and a second order Maxwell-Chern-Simons (MCS) theory. Both theories represent one massive polarization state in $D = 2+1$ spacetime of helicity $+1$ or -1, depending on the sign of the Chern-Simons coefficient. As expected from the lack of gauge invariance of the SD theory, the map between the theories $f_\mu \leftrightarrow \frac{\epsilon_{\mu\nu\gamma}\partial^\nu A^\gamma}{m}$ is invariant under gauge transformations of the Maxwell-Chern-Simons fundamental field A^γ and holds at classical and quantum level including gauge invariant correlation functions [8]. The natural addition of a Maxwell term in the self-dual model however, spoils the simplicity of the duality relation between the now called generalized self-dual (GSD) model and its possible gauge invariant dual theory. A direct generalization of the master action approach leads, quite surprisingly, to a gauge theory [9] which now includes a ghost mode in the spectrum, so the existence of a master action which interpolates between two theories does not guarantee spectrum equivalence *a priori*. As explained in [10] if we insist in the spectrum equivalence a new master action can be suggested which leads however, to a non-local vector theory. It is seems that we have glanced the old problem of formulating massive theories in a gauge invariant way. In this work we show that by introducing an auxiliary vector field to lower the Maxwell term to first order we are able to define another master action which naturally interpolates between the GSD model and a well defined gauge invariant local theory which corresponds to a couple of non-interacting Maxwell-Chern-Simons theories of opposite helicities, henceforth called 2MCS. It turns out that the

GSD model and the 2MCS models were know to be related for a long time [11, 12]. In particular, it has been shown in [13, 14] that the two MCS models could be soldered into the GSD model. Our results are complementary to the soldering procedure and work in the opposite direction like the canonical transformations of [15, 16]. In the next section we quickly review previous master action attempts and suggest a new master action and a generating functional which allows us to compare correlation functions in both theories. In section III we concentrate on the classical equivalence, clarifying how a theory of two non-interacting vector fields can be shown to be physically equivalent to the GSD model which contains only one vector field. We also comment in section III on the coupling to matter fields. In section IV we draw our conclusions.

2. Master action and quantum equivalence

Let us first present the GSD model which might be called also a Maxwell-Chern-Simons-Proca model[1]:

$$\mathcal{L}_{\rm GSD} \, = a_0 f^\mu f_\mu + a_1 \epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma - \frac{a_2}{2} F_{\mu\nu}(f) F^{\mu\nu}(f) \tag{2.1}$$

For $a_2 = 0$ we recover the self-dual model of [7]. Due to unitarity reasons we need to have [17] (see also [10]) $a_0 \geq 0$ and $a_2 \geq 0$. The constants a_i are otherwise arbitrary. Henceforth we assume that a_0 and a_2 are definite positive. We can write down the equations of motion of (2.1) in a self-dual form generalizing the definition of duality transformations:

$$f_\mu = \frac{1}{a_0} \left(a_1 E_{\mu\nu} - a_2 \Box \theta_{\mu\nu} \right) f^\nu \equiv f^*. \tag{2.2}$$

We have defined the differential operators:

$$E_{\mu\nu} = \epsilon_{\mu\nu\gamma} \partial^\gamma \quad , \quad \Box \theta_{\mu\nu} = \Box g_{\mu\nu} - \partial_\mu \partial_\nu \tag{2.3}$$

Note the useful identities $E_{\mu\nu} E^{\nu\alpha} = -\Box \theta_\mu^\alpha$, $E_{\mu\nu} \theta^{\nu\alpha} = E_\mu^\alpha$ and $\theta_{\alpha\beta} \theta^{\beta\gamma} = \theta_\alpha^\gamma$. From (2.2) we can derive the existence of two massive modes in the self-dual field: $\left(\Box + m_+^2\right)\left(\Box + m_-^2\right)$ $f_\mu = 0$, where

$$2m_\pm^2 \, = b^2 + 2a \pm \sqrt{(b^2 + 2a)^2 - 4a^2} \quad , \tag{2.4}$$

with $a = a_0/a_2$, $b = a_1/a_2$. Those massive physical particles can be confirmed by checking the poles and the corresponding signs of the residues of the propagator. The expression (2.4) can be inverted for the ratios a, b:

$$a_0 = a_2 \, m_+ m_- \quad ; \quad a_1 = a_2 \, (m_+ - m_-) \quad . \tag{2.5}$$

There is a sign freedom in the solution for a_1 but we choose it to be positive for definiteness. Henceforth we can describe the GSD model as defined by the three parameters a_2, m_+, m_- according with (2.1) and (2.5). This is a more physical notation which makes clear, in

[1] Comparing to [10] we have slightly changed our notation $a_2 \to -a_2$ but we still use $g_{\mu\nu} = (+, -, -)$.

particular, that the mass split comes from parity breaking. If $a_1 = 0$ we have the Maxwell-Proca theory with two particles with opposite helicities ± 1 but with degenerate mass $m_+ = m_-$.

In order to suggest a new master action which would produce a local gauge theory dual to (2.1) we recall previous attempts. Both suggestions of [9] and [10] can be cast in the form of a gauge invariant second order master equation:

$$\begin{aligned}\mathcal{L} &= a_0 f^\mu f_\mu + a_1 \epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma - \frac{a_2}{2} F_{\mu\nu}(f) F^{\mu\nu}(f) \\ &- b_1 \epsilon_{\alpha\beta\gamma}(A^\alpha - f^\alpha)\partial^\beta (A^\gamma - f^\gamma) + \frac{b_2}{2} F_{\mu\nu}(A-f) F^{\mu\nu}(A-f) \end{aligned} \tag{2.6}$$

The proposal of [9] corresponds to $(b_1, b_2) = (a_1, a_2)$. The advantage of this choice is that all quadratic terms in the self-dual field except the first one on the right-handed side of (2.6) are cancelled, which gives rise to a local gauge theory upon integration in the self-dual field. However, the theory thus obtained contains a ghost pole in the propagator, which is in agreement with the predictions of [18]. The presence of the ghost could have been foreseen also from the fact that after a trivial shift $A_\mu \to A_\mu + f_\mu$, the theory (2.6) can be written as a GSD model decoupled from a Maxwell-Chern-Simons Gauge theory where the coefficient of the Maxwell term appears with the wrong sign. The integration on the self-dual field reintroduces, as explained in [10], the ghost mode in the resulting gauge theory. So the message is clear, i.e., we better mix the self-dual and the gauge field through a Lagrangian density which has no particle content thus guaranteeing the spectrum match of both gauge and non-gauge theories. This the case of the choice $(b_1, b_2) = (a_1, 0)$ where the mixing comes only from the topological Chern-Simons term which contains no physical degree of freedom. Indeed, this choice leads to a gauge theory [10] equivalent to the GSD model, up to contact terms in the correlation functions, and with the same massive poles $k^2 = m_\pm^2$ without extra particles in the spectrum. Due to the non-cancelation of the quadratic terms in the self-dual field which involve derivatives, we pay the price of loosing locality upon integration on the self-dual field. A key ingredient lacking in (2.6) but present in the original proposal of a master action in [5] is to start with a first order Lagrangian. It is easy to reduce the Maxwell term to first order by using an auxiliary vector field (g_μ), such that we are led to the following suggestion:

$$\begin{aligned}\mathcal{L}_{\rm Master} &= a_0 f^\mu f_\mu + a_1 \epsilon_{\alpha\beta\gamma} f^\alpha \partial^\beta f^\gamma + g^\mu g_\mu + \sqrt{a_2}\, g_\mu \epsilon^{\mu\alpha\beta} F_{\alpha\beta}(f) + f_\mu j^\mu \\ &- a_1 \epsilon_{\alpha\beta\gamma}(\tilde{A}^\alpha - f^\alpha)\partial^\beta(\tilde{A}^\gamma - f^\gamma) - \sqrt{a_2}\,(\tilde{B}_\mu - g_\mu)\epsilon^{\mu\alpha\beta} F_{\alpha\beta}(\tilde{A} - f) \end{aligned} \tag{2.7}$$

For $a_2 = 0$ the auxiliary field g_μ decouples and we recover the master action of [5] plus a source term for the self-dual field that we have introduced for future use. After the shifts $\tilde{A}_\mu \to \tilde{A}_\mu + f_\mu$ and $\tilde{B}_\mu \to \tilde{B}_\mu + g_\mu$ we end up with the GSD model decoupled from a trivial topological theory for the fields $\tilde{A}_\mu$ and $\tilde{B}_\mu$ with no particle content. If we finally integrate over $\tilde{A}_\mu, \tilde{B}_\mu$ and g_μ in the path integral we derive the GSD model (2.1) plus a source term:

$$\mathcal{Z}(j) = \int \mathcal{D}f^\nu \mathcal{D}g^\nu \mathcal{D}\tilde{A}^\nu \mathcal{D}\tilde{B}^\nu \, e^{i\int d^3x \mathcal{L}_{\rm Master}} = \int \mathcal{D}f^\nu \, e^{i\int d^3x\,(\mathcal{L}_{\rm GSD} + j_\mu f^\mu)} \tag{2.8}$$

On the other hand, since in the master action (2.7) there are no quadratic terms in the fields f_μ and g_μ involving derivatives , they can be easily integrated over such that we are left with a local gauge theory corresponding to a couple of interacting Maxwell-Chern-Simons models plus source dependent terms:

$$\begin{aligned}\mathcal{Z}(j) &= \int \mathcal{D}f^\nu \mathcal{D}g^\nu \mathcal{D}\tilde{A}^\nu \mathcal{D}\tilde{B}^\nu \, \exp i \int d^3x \mathcal{L}_{\text{Master}} \\ &= \int \mathcal{D}\tilde{A}^\nu \mathcal{D}\tilde{B}^\nu \, \exp \, i \int d^3x \left[\tilde{\mathcal{L}}(\tilde{A},\tilde{B}) - \frac{j_\mu j^\mu}{4a_0} - \frac{j^\mu \epsilon_{\mu\nu\gamma} F^{\nu\gamma}(a_1\tilde{A} + \sqrt{a_2}\tilde{B})}{2a_0} \right] \end{aligned} \tag{2.9}$$

where

$$\tilde{\mathcal{L}}(\tilde{A},\tilde{B}) = -\frac{1}{2a_0} F^2_{\alpha\beta}(a_1\tilde{A}+\sqrt{a_2}\tilde{B}) - \frac{a_2}{2} F^2_{\alpha\beta}(\tilde{A}) - a_1 \epsilon_{\mu\nu\gamma} \tilde{A}^\mu \partial^\nu \tilde{A}^\gamma - 2\sqrt{a_2}\epsilon_{\mu\nu\gamma}\tilde{A}^\mu \partial^\nu \tilde{B}^\gamma \tag{2.10}$$

After appropriate field redefinitions we can rewrite $\tilde{\mathcal{L}}(\tilde{A},\tilde{B})$ as a couple of non-interacting Maxwell-Chern-Simons theories. For instance, using

$$\begin{aligned}\tilde{A}_\mu &= \frac{1}{\sqrt{2a_2(m_+ + m_-)}} \left(\sqrt{m_+}A_\mu - \sqrt{m_-}B_\mu\right) \\ \tilde{B}_\mu &= \frac{-1}{\sqrt{2(m_+ + m_-)}} \left(\sqrt{m_+^3}A_\mu + \sqrt{m_-^3}B_\mu\right)\end{aligned} \tag{2.11}$$

We have

$$\tilde{\mathcal{L}}(\tilde{A},\tilde{B}) = \mathcal{L}_{2MCS}(A,B) = -\frac{F^2_{\alpha\beta}(A)}{4} + \frac{m_+}{2}\epsilon_{\mu\nu\gamma}A^\mu\partial^\nu A^\gamma - \frac{F^2_{\alpha\beta}(B)}{4} - \frac{m_-}{2}\epsilon_{\mu\nu\gamma}B^\mu\partial^\nu B^\gamma \tag{2.12}$$

The field redefinitions (2.11) are not unique but the other possible choices also lead to the same non-interacting Chern-Simons theories (2.12) up to trivial field rescalings. In terms of the new fields we can rewrite (2.9), up to a trivial constant Jacobian, as follows:

$$\mathcal{Z}(j) = \int \mathcal{D}A^\nu \mathcal{D}B^\nu \, \exp\left\{ i \int d^3x \left[\mathcal{L}_{2MCS}(A,B) - \frac{j_\mu j^\mu}{4a_2\, m_+ m_-} + j^\mu C_\mu \right]\right\} \tag{2.13}$$

Where we have defined the gauge invariant combination

$$C_\mu = -\frac{1}{2a_0}\epsilon_{\mu\nu\gamma}F^{\nu\gamma}(a_1\tilde{A} + \sqrt{a_2}\tilde{B}) = \frac{\epsilon_{\mu\nu\gamma}\partial^\nu}{\sqrt{a_2(m_+ + m_-)}}\left(\frac{A^\gamma}{\sqrt{m_+}} + \frac{B^\gamma}{\sqrt{m_-}}\right) \tag{2.14}$$

Deriving (2.13) and (2.8) with respect to the sources we have the equivalence of correlation functions:

$$\langle f_{\mu_1}(x_1)\cdots f_{\mu_N}(x_N)\rangle_{\text{GSD}} = \langle C_{\mu_1}(x_1)\cdots C_{\mu_N}(x_N)\rangle_{2MCS} + \text{contact terms} \quad . \tag{2.15}$$

Where the contact terms (delta functions) come from the quadratic term in the sources appearing in (2.13). As expected from the fact that the GSD model is not a gauge theory, we have identified correlation functions of the self-dual field with correlation functions of a gauge invariant object in the 2MCS theory with no need of introducing an explicit gauge condition. By examining the propagators of the fields A_μ and B_μ in the 2MCS model we notice that that both have a pole at momenta $k^2 = 0$ which represents in fact a non-propagating mode (vanishing residue [18]) and a physical pole at $k^2 = m_+^2$ and $k^2 = m_-^2$ respectively. Therefore the spectrum of the 2MCS and the GSD models are equivalent as expected. However, it is rather disturbing for a complete proof of equivalence of such models that the correlation functions of the self-dual field can be written in terms of correlation functions of only one specific linear combination of A_μ and B_μ fields which are on their turn independent and non-interacting fields and can not be written of course in terms of just one linear combination. It is natural to ask whether correlation functions of both fields A_μ, B_μ can be in general calculated from the GSD theory. In order to answer that question we define a new generating function below which allows the computation of the relevant gauge invariant correlators of the 2MCS theory:

$$\mathcal{Z}(j_A, j_B) = \int \mathcal{D}f^\nu \mathcal{D}g^\nu \mathcal{D}\tilde{A}^\nu \mathcal{D}\tilde{B}^\nu \, \exp\left\{ i \int d^3x \left[\mathcal{L}_{\text{Master}} + j_A^\mu F_\mu(A) + j_B^\mu F_\mu(B) \right] \right\} \tag{2.16}$$

$$= \int \mathcal{D}f^\nu \mathcal{D}g^\nu \mathcal{D}\tilde{A}^\nu \mathcal{D}\tilde{B}^\nu \, \exp\left\{ i \int d^3x \left[\mathcal{L}_{\text{Master}} + r \left(m_- \bar{j}_A^\mu - m_+ \bar{j}_B^\mu \right) F_\mu(\tilde{A}) - \frac{r}{\sqrt{a_2}} \left(\bar{j}_A^\mu + \bar{j}_B^\mu \right) F_\mu(\tilde{B}) \right] \right\} \tag{2.17}$$

Where we have introduced the constant $r = \sqrt{2a_2/(m_+ + m_-)}$, the dual field strength $F_\mu(A) = \epsilon_{\mu\nu\gamma}\partial^\nu A^\gamma$ and the redefined sources $\bar{j}_A^\mu = j_A^\mu/\sqrt{m_+}$; $\bar{j}_B^\mu = j_B^\mu/\sqrt{m_-}$. In obtaining (2.17) from (2.16) we have inverted the linear transformations (2.11). Since the 2MCS model follows from the master action by integrating over $\mathcal{D}f^\nu \mathcal{D}g^\nu$ it is clear that j_A^μ and j_B^ν are the correct sources for computing correlation functions of $F^\mu(A)$ and $F^\nu(B)$ in the 2MCS theory respectively. Now if we integrate over $\mathcal{D}g^\nu \mathcal{D}\tilde{A}^\nu \mathcal{D}\tilde{B}^\nu$ we deduce:

$$\mathcal{Z}(j_A, j_B) = \int \mathcal{D}f^\nu \exp i \int d^3x \left[\mathcal{L}_{\text{GSD}} - r\left(\bar{j}_A^\mu + \bar{j}_B^\mu\right) \Box\theta_{\mu\nu} f^\nu - \frac{1}{4(m_+ + m_-)} F_{\alpha\beta}^2 \left(\bar{j}_A^\mu + \bar{j}_B^\mu\right) + r\left(m_- \bar{j}_A^\mu - m_+ \bar{j}_B^\mu\right) \epsilon_{\mu\nu\gamma}\partial^\nu f^\gamma - \frac{1}{2}\left(\bar{j}_A^\mu - \bar{j}_B^\mu\right) \epsilon_{\mu\nu\gamma}\partial^\nu \left(\bar{j}_A^\gamma + \bar{j}_B^\gamma\right) \right] \tag{2.18}$$

In conclusion, we can indeed calculate correlation functions of the 2MCS theory from the GSD model. Explicitly,

$$\begin{aligned} &\langle F_{\mu_1}[A(x_1)] \cdots F_{\mu_N}[A(x_N)] F_{\nu_1}[B(y_1)] \cdots F_{\nu_N}[B(y_M)] \rangle_{2MCS} \\ &= \hat{T}_{\mu_1\alpha_1}(m_-, x_1) \cdots \hat{T}_{\mu_N\alpha_N}(m_-, x_N) \hat{T}_{\nu_1\beta_1}(-m_+, y_1) \cdots \hat{T}_{\nu_M\beta_M}(-m_+, y_M) \times \\ &\times \left\langle f^{\alpha_1}(x_1) \cdots f^{\alpha_N}(x_N) f^{\beta_1}(y_1) \cdots f^{\beta_M}(y_M) \right\rangle_{\text{GSD}} + \text{contact terms} \end{aligned} \tag{2.19}$$

Where $\hat{T}_{\alpha\beta}(m, x) = -\left(r/\sqrt{|m|}\right)\left(\Box\theta_{\alpha\beta} + mE_{\alpha\beta}\right)_x$. One can check, as we have done, the correctness of (2.18) and (2.19) by calculating two point functions in the 2MCS theory from the self-dual propagator in the GSD theory plus the contact terms. In particular, the contact terms are such that one verifies the trivial result $\langle F_\mu[A(x)] F_{\nu_N}[B(y)] \rangle_{2MCS} = 0$.

The results (2.15) and (2.19) demonstrate the quantum equivalence of the gauge invariant sector of the 2MCS model to the GSD model. The equivalence holds up to contact terms which vanish for non-coinciding points.

3. Classical equivalence and generalized self-duality

From the master action $\mathcal{L}_{\text{Master}}(f, g, \tilde{A}, \tilde{B})$ given in (2.7) we have the following equations of motion:

$$d\left(\tilde{A} - f\right) = 0 \rightarrow \tilde{A}_\mu = f_\mu + \partial_\mu \tilde{\phi} \tag{3.1}$$

$$d\left(\tilde{B} - g\right) = 0 \rightarrow \tilde{B}_\mu = g_\mu + \partial_\mu \tilde{\psi} \tag{3.2}$$

$$g_\mu = \sqrt{a_2} E_{\mu\alpha} \tilde{A}^\alpha \tag{3.3}$$

$$f_\mu = \frac{1}{a_0} E_{\mu\nu} \left(a_1 \tilde{A}^\nu + \sqrt{a_2} \tilde{B}^\nu\right) \quad , \tag{3.4}$$

with $\tilde{\phi}, \tilde{\psi}$ arbitrary functions. The equations (3.1), (3.2) and (3.3) may be used to eliminate the fields $\tilde{A}_\mu, \tilde{B}_\mu$ and g_μ in terms of f_μ which becomes the only independent degree of freedom. In this case (3.4) becomes the generalized self-dual equation $f_\mu = f_\mu^*$ as in (2.2). On the other hand, we could have used (3.1) and (3.2) to write f_μ and g_μ in terms of $\tilde{A}_\mu$ and $\tilde{B}_\mu$ respectively. Accordingly, plugging back the result in (3.3) and (3.4) we derive:

$$\tilde{B}_\mu = \sqrt{a_2} E_{\mu\nu} \tilde{A}^\nu + \partial_\mu \tilde{\psi} \tag{3.5}$$

$$\tilde{A}_\mu = \partial_\mu \tilde{\phi} + \frac{1}{a_0} E_{\mu\nu} \left(a_1 \tilde{A}^\nu + \sqrt{a_2} \tilde{B}^\nu\right) = \partial_\mu \tilde{\phi} + C_\mu \tag{3.6}$$

$$= \partial_\mu \tilde{\phi} + \frac{1}{a_0} \left(a_1 E_{\mu\nu} - a_2 \Box \theta_{\mu\nu}\right) \tilde{A}^\nu = \partial_\mu \tilde{\phi} + \tilde{A}_\mu^* \tag{3.7}$$

Where the combination C_μ is the same one defined in (2.14). Since the generalized duality transformation is such that $\left(\partial_\mu \tilde{\phi}\right)^* = 0$, it is clear from (3.7) that $\tilde{A}_\mu^* = \left(\tilde{A}_\mu^*\right)^*$ and using $\tilde{A}_\mu^* = C_\mu$ we deduce the self-dual equation $C_\mu = C_\mu^*$. Therefore, we can say that the map below holds at quantum and classical level:

$$f_\mu \Leftrightarrow C_\mu \tag{3.8}$$

In summary, on one hand we have the equations of motion of the first order version of the GSD model which can be written as $g_\mu = \sqrt{a_2} E_{\mu\alpha} f^\alpha$ and $f_\mu = f_\mu^*$. On the other hand, the equation (3.5) teaches us that the combination $\tilde{B}_\mu$ can be eliminated in terms of $\tilde{A}_\mu$ in parallel to the elimination of g_μ as function of the self-dual field, while the dynamical degree of freedom $\tilde{A}_\mu$ satisfies $\tilde{A}_\mu^* = \left(\tilde{A}_\mu^*\right)^*$ which is equivalent to $C_\mu = C_\mu^*$ and therefore completes the analogy with the GSD model. So in both theories we have only one independent dynamical vector field which satisfies a self-duality condition. Thus, we can say that both theories are generalized versions of the self-dual model of [7]. From the point of view of the non-interacting MCS fields A_μ and B_μ it is quite surprisingly that there is only one independent dynamical vector field. The reader may find useful to obtain the equations (3.5) and (3.6) directly from the 2MCS theory as we do next in order to clarify this point. Minimizing $\mathcal{L}_{2MCS}$ we have:

$$E_{\mu\nu}\left(m_- B^\nu - E^{\nu\alpha}B_\alpha\right) = 0 \;\rightarrow\; B^\nu = \frac{E^{\nu\alpha}B_\alpha}{m_-} + \partial^\nu\psi \tag{3.9}$$

$$E_{\mu\nu}\left(m_+ A^\nu + E^{\nu\alpha}A_\alpha\right) = 0 \;\rightarrow\; A^\nu = -\frac{E^{\nu\alpha}A_\alpha}{m_+} + \partial^\nu\phi \tag{3.10}$$

The general solutions (3.9) and (3.10) lead to $-m_+^{3/2}A^\nu - m_-^{3/2}B^\nu = E^{\nu\alpha}(\sqrt{m_+}A_\alpha - \sqrt{m_-}B_\alpha) - \partial^\nu\left(m_+^{3/2}\phi + m_-^{3/2}\psi\right)$ which is equivalent to equation (3.5), i.e., $\tilde{B}^\nu = \sqrt{a_2}E^{\nu\alpha}\tilde{A}_\alpha + \partial^\nu\tilde{\psi}$. This confirms that we can treat $E^{\nu\alpha}\tilde{A}_\alpha$ as the only independent dynamical vector field in the 2MCS model. Analogously, from (3.9) and (3.10) we have $\sqrt{m_+}A_\mu - \sqrt{m_-}B_\mu = -E_{\mu\alpha}\left(A^\alpha/\sqrt{m_+} + B^\alpha/\sqrt{m_-}\right) + \partial_\mu(\sqrt{m_+}\phi - \sqrt{m_-}\psi)$ from which we can derive $\tilde{A}^\mu = E^{\mu\alpha}\left[\tilde{A}_\alpha(m_+ - m_-) + \tilde{B}_\alpha/\sqrt{a_2}\right]/(m_+m_-) + \partial_\mu\tilde{\phi}$ which is equivalent to equation (3.6) and consequently we deduce the generalized self-duality equation $C_\mu = C_\mu^*$ with $C_\mu = A_\mu^*$. The quantities $\tilde{\psi}, \tilde{\phi}$ are of course linear combination of ψ and ϕ.

At last, we briefly comment on the coupling of the GSD model to matter and its dual gauge theory. We notice that the GSD model is not a gauge theory, so there is no reason to minimally couple it to U(1) matter. In particular, it is natural, see comments in [19], to consider a linear coupling of the self-dual field to a U(1) matter current which may represent fermions or bosons (scalars). By repeating the steps which have taken us from the GSD to the 2MCS model and substitute j^μ by $j^\mu_{\rm matter}$ it is easy to verify that we have the following duality relation when we include matter:

$$\begin{aligned}\mathcal{L}_{\rm GSD}(f) + \mathcal{L}_{\rm matter} + f_\mu j^\mu_{\rm matter} \Leftrightarrow \mathcal{L}_{2MCS}(A,B) + \mathcal{L}_{\rm matter} - \frac{j^\mu_{\rm matter}j_{\mu\,\rm matter}}{4a_0} \\ + \frac{j^\mu_{\rm matter}\epsilon_{\mu\nu\gamma}\partial^\nu}{\sqrt{a_2(m_+ + m_-)}}\left(\frac{A^\gamma}{\sqrt{m_+}} + \frac{B^\gamma}{\sqrt{m_-}}\right)\end{aligned} \tag{3.11}$$

Therefore, the dual gauge theory contains a Thirring-like term plus a non-minimal coupling of the Pauly-type as in [20, 9]. Only a specific gauge invariant linear combination of the Chern-Simons fields couples to the matter current. We interpret the appearance of non-renormalizable interactions in the dual gauge theory as a consequence of the bad ultraviolet behavior of the self-dual propagator, which becomes a constant for large momenta in spite of the presence of the Maxwell term.

4. Conclusion

We have suggested here in a systematic way a new master action which correctly interpolates between a generalized self-dual model (GSD) and its dual gauge theory consisting of a couple of non-interacting Maxwell-Chern-Simons fields of opposite helicities (2MCS) which is local and ghost free as opposed to previous proposals. The master action suggested here, by construction, assures that the dual theories have the same spectrum which is not a general feature of the master action approach as explained in [10]. Another key ingredient was the reduction of the second order Maxwell term to first order by means of an auxiliary vector field g_μ besides the dynamical self-dual field f_μ. This approach allowed a natural parallel with the two fields of the 2MCS theories thus, explaining the apparent doubling of fields on one side of the duality. It turns out that both GSD and 2MCS models have a superfluous vector field which can be eliminated in favor of a gauge invariant dynamical vector field whose equation of motion can be written as a generalized self-duality condition.

Furthermore, we have found a map, see (3.8), between the dual theories which holds at classical and quantum level. In the opposite direction one can also calculate the relevant gauge invariant correlators of the 2MCS theory from the GSD model plus contact terms. Our work demonstrates a complete equivalence between those models and, differently from [16], no explicit gauge condition has been fixed.

It is possible (under investigation now) that other soldered theories, see [21] for more examples, can be similarly "unsoldered" as we have done here. In particular, it is tempting to investigate by an interpolating master action the doubling of fields in the electric-magnetic duality invariant Schwarz-Sen model [22] in $3+1$ dimensions.

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